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# Affectation of emergency vehicles in Rescue Centers under random demand

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**Abstract:** The aim of this paper is to improve the operational response of the Fire and Rescue Services (SDIS). More precisely, the allocation of emergency vehicle (EV) to Rescue Center (RC) in department(France) is studied. The problem is to allocate the capacity (number of EV by RC) under random apparition of accidents and random durations. A mathematical model based on the notion of simultaneous requirements of EV is proposed. This notion allows us to dispense with the time dimension and just focus on the probability to have simultaneous required quantity of EVs. Hence, the problem can be formulated as a kind of transportation problem under random demand. An approximation of the model is done using the simulation approach and an algorithm to find the optimal solution of approximate model is proposed.

*Keywords:* Rescue Center, minimization problems, Transport.

## 1. INTRODUCTION

In France, a public service called SDIS (Fire and Rescue Services) exists in each department. A fire fighting unit (professional as well as voluntary), technical and administrative staff make up this service. Its main missions are as follows: Operations, Preparation and Prevention. The SDIS prevents all risks for civil security, like accidents, damage and natural hazards ; it prepares protective measures and organises special equipment and also fights fires of any kind, offers emergency aid.

The SDIS is composed by Rescue Centers (RCs), so the department is divided into sectors and each town(1) is protected by one or several RCs (which can be located in the neighbour town) and (2) linked to all RCs of department by priority list calling "Deployment plan". The first RC of deployment plan, called "first call center", are those normally called to intervene on the town. The second ones on the deployment plan, called "second call center", are those called in case of the unavailability of the first call center or can be required for help in the event of an important disaster and so on to the last RC. The deployment plan are built automatically using the time needed to go from RC to the town: first center is the closer second center is the second closer and so on. The aim of a SDIS is, for 80% of interventions, to have a set of centers which are able to intervene within 10 and 20 minutes (in function of predefined zones) in the case of human rescue, fire and various operations. In this context, a strong area of improvement is to parametrize the capacity management of each RC under random demand.

In the literature, models have been proposed to optimize the static and deterministic covering problem in facility location (see the reviews Farahani et al. 2012, Brotcorne

et al. 2003 and Dasin et al. 2004). They models are based on:

- A Location Set Covering Problem (LSCP) is studied by (Tonegas et al. 1971) to minimize the number of facilities (centers) to cover the set of demand points short of the indicated coverage.
- A Maximal Covering Location Problem (MCLP) and a Modular Capacitated Maximal Covering Location Problem (MCMCLP) are studied by (Church and Reville 1974) to determine the number of facilities required to maximize the number of demand points served by limited fixed number of facilities.

In these models all facilities are independent, are considered as a resource center (ie. a facility is a fire station). They have infinite capacity or are considered as a single resource (ie. a facility is an ambulance) and they are always considered available. To take into account the availability of resources, probabilistic models have been proposed which determine the location to maximize, with a given probability, the covered population that can be satisfied by an available facility (Daskin, 1983, Reville and Hogan, 1989). To introduce capacity constraints, a capacitated MCLP has been proposed with a fixed capacity level of the facility for each potential facility site (Current and Storbeck 1988) and more recently, (Yin et al. 2012) propose a Modular Capacitated MCLP with different capacity levels (varied numbers of stationed emergency vehicles). Moreover in these two models, it is not specified which demand can be covered by which facility. To cope with this limitation, Generalized MCPL (GMCP) models (Berman and Krass, 2002), Gradual coverage models (Berman et al. 2010) or Backup Cover in Location Problems models BCLP (Hogan and Reville, 1986) have been developed.

This literature focus on localization of facilities and the first center allocation with deterministic demand. In our case the localization of facilities (RCs), the deployment plans (hence the ordered list of facilities which can cover the demands) and the total number of resources (EVs) are known. Besides, the demand is stochastic and the problem is to determine the allocation of the resources (EVs) to the different facilities (RCs).

In our case, we consider a set of given facilities (RC). In each facility a set of resources (EVs) is available to response to the demand. A demand is not satisfied by a given facility but by a priority list of facilities (deployment plans). The total number of resources is known and the problem consists in a re-allocation of this given number of resources to the different utilities. Thus this problem can be considered as a specific dynamic facility location problem where the facility location is known and a given number of resources have to be re-allocated to these facilities.

Furthermore, in the field of ambulance relocation problems (Brotcorne, et al. 2003), the location problem mostly consists in a covering facility location problem which consider the ambulances as the facilities to be located. In addition, the relocation problem is mostly treated as a real time problem.

Our problem can be situated between these strategic covering facility location problem and the real time relocation problem. It is closed to the problem addressed by (Beraldi and al. 2004) who developed a stochastic programming model with probabilistic constraints which aims to solve both location of facilities (which are emergency service sites) and allocation of emergency vehicles to each site.

Our study of data of department "Haute-Garonne" (France) and an analysis of department "Ille-et-Vilaine" (SDACR 35) show that the probability that a number of EVs are needed in the simultaneously follows a Poisson distribution. Schmauch (2007) proposes an heuristic which consists too affected the EVs to RCs using the probability of simultaneity requirement of EV of the first center. This approach does not take into account others RCs of deployment plan which are crucial to estimate the maximal time needed to arrive on the accident. In fact, The analysis of the activity of a given RC is not pertinent to know the best size of a given sector: a center can often be engaged in other sectors (sectors not belong to the set of town which have RC as first center). Conversely, a sector may be frequently covered by the second call center or the third. This problem is a kind of transportation problem (explaining in section 3.8) under stochastic demand. In the literature, the problem of determining the transportation quantities under stochastic demand has been studied by (Szwarc, 1964). In our case the transportation quantities are determined under crisp demand, but the capacity of RCs under stochastic demand that is different. As far as we know, our problem has not be studied in the literature.

In this paper, we propose a model to affect the EVs to RCs based on the simultaneity of requirement of EV taking into account the notion of deployment plans. The aim is to minimize the degradation of the case where all accidents are treated by the first center of deployment plan. In other worlds, we want to minimize the degradation of the opti-

mal solutions of this problem without capacity constraints. Since we have random simultaneous requirements and we want to minimize the expected value of the degradation. The rest of the paper is organized as follow: Firstly, we describe the problem (section 2). The mathematical model is given in section 3. Section 4 shows the optimization method which is used to resolve this problem. Finally, as conclusion, some perspectives of future research are given.

## 2. PROBLEM DESCRIPTION

In this paper, we focus on the optimization of the EV number by RS. We note that both the sum of EVs and the set of RC are given.

### 2.1 Deployment plans

Each town is associated to a deployment plan which gives the priorities of intervention of each RC. Usually, the nearest to the furthest RC. For example, the town 2 (striped area with fine lines in figure 1) has the following deployment plan: the RC of the town 3 then the RC of the town 1 then the RC of the town 6; the town 3 (area composed from horizontal dashed lines): the RC of the town 3 then the RC of the town 6 then the RC of the town 1: We note that the first RC in a deployment plan is considered as the first call center.

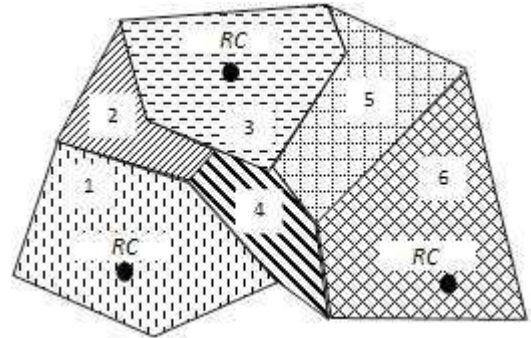


Fig. 1. Deployment plans

### 2.2 Operational coverage

The primary objective of fire fighting unit is to arrive as soon as possible to the accident place. In an unconstrained resource context, it would have enough vehicles in each RC to satisfy the demand of all towns which are this RC as a first call center. We note that the first call center in a deployment plan is the nearest RC to the accident place. Under resources constraint, the operational coverage can be formulated as follows: To minimize the mean number of interventions from RC when these RCs are not first call centers. Our aim is also to reassignment vehicles in a given area. We supposed that if the requirement in a given area is not satisfied by available vehicles in this area, other vehicles from other areas will be called in reinforcement.

### 2.3 Simultaneous requirement of EV

In fact, the requirement of EVs in each RC depends on the number of accidents that happened simultaneously in towns which have this RC as a first call centre. This simultaneity of requirements is due to the appearance of

one or several accidents that need one or several EVs during the time that one or several others EV are occupied by one or more other accidents.

Otherwise, the number of simultaneous requirements (noted  $n$ ) is a random number since both accidents happening and durations of use of EVs are random.

More precisely, a study of data of RCs (SDARC 35, 2010) shows that the probability distribution of simultaneous requirements for a given deployment plan (noted  $\tilde{d}_\pi$ ) follows a Poisson distribution:

$$P([\tilde{d}_\pi = n_\pi]) = \exp(-\gamma_\pi) \times \frac{\gamma_\pi^{n_\pi}}{n_\pi!}$$

where  $\gamma_\pi = \frac{N_\pi \times t_\pi}{8760}$ ,  $N_\pi$  is the number of accidents during one year on the municipalities under the same deployment plan  $\pi$ ,  $t_\pi$  the average of duration for the municipalities under the same deployment plan  $\pi$  and 8760 the total number of hours by year.

### 3. MATHEMATICAL FORMULATION

Firstly, a mathematical formulation is proposed to allocate EVs to a RC for a given deployment plan and for a given number of simultaneous requirements. Then this model will be extended to the case of re-affectation of EVs to all RCs. Finally, the model of re-affectation of EVs under random simultaneous requirements will be presented.

Let  $\mathbb{C}$  the set of RCs,  $\text{card}(\mathbb{C}) = c$  and  $\mathbb{P}$  the set of deployment plans. Each deployment plan is denoted  $\pi = (\pi(1), \pi(2), \dots)$  where  $\pi(1)$  is the first RC called,  $\pi(2)$  the second RC called if the first one has no available EV etc.

#### 3.1 Data

Assume that an accident took place in the town whose deployment plan is  $\pi = (\pi(1), \pi(2), \dots, \pi(i), \dots)$ . For  $i \in \mathbb{C}$ ,  $j \in \mathbb{C}$  and  $\pi \in \mathbb{P}$ :

- $t_{i,\pi}$ : the average time needed for a EV from the RC  $i$  to arrive to the scene of the accident. if the RC  $i$  is the first called center,  $t_{i,\pi} = t_{\pi(1),\pi}$ ,
- $a_{i,\pi}$ : the average lost time if a EV from a RC  $i$  is used. It is equal to  $t_{i,\pi} - t_{\pi(1),\pi}$ .  $a_{i,\pi} = 0$  if  $\pi(1) = i$  and  $a_{i,\pi} < a_{j,\pi}$  if  $i$  precedes  $j$  in the deployment plan  $\pi$ ,
- $a_{e,\pi}$ : the penalty to use a EV outside the considered department,
- $d_\pi$ : the number of simultaneous required EVs for a deployment plan  $\pi$ .
- $Q$ : the total number of available EVs on the studied department.

#### 3.2 Variables

All considered variables are positive integers.

- $h_{i,\pi}$ : the number of EVs from the RC  $i$  allocated to the deployment plan  $\pi$ ,
- $h_{e,\pi}$ : the number of EVs outside the considered department allocated to the deployment plan  $\pi$
- $K_i$ : the capacity of the RC  $i$  in terms of the number of EVs.

#### 3.3 Model under known capacities and fixed simultaneous required EV

To help understand this problem, a sub problem under simplifications conditions is presented: (1) the capacity of each RC is given and is not variable to be determined by the system and (2) simultaneous required EVs of deployment plans are known.

Under these conditions, the problem is equivalent to a classical transportation problem where each demand of a customer is replaced by the simultaneous required EVs of the deployment plans ( $d_\pi$ ), the production is replaced by the capacity of the RC ( $K_i$ ) and the transportation quantities are replaced by the EVs allowed from RCs to deployment plans ( $h_{i,\pi}$ ,  $h_{e,\pi}$ ). The costs of transportation are the lost times ( $a_{i,\pi}$ ).

So, this problem can be formulated as follow:

$$\begin{aligned} \min_{\{h_{i,\pi}\}_{i \in \mathbb{C}, \pi \in \mathbb{P}}} \quad & F(h_{i,\pi}) = \sum_{\pi \in \mathbb{P}} \sum_{i \in \mathbb{C}} a_{i,\pi} h_{i,\pi} \\ \text{s.t.} \quad & \\ (a) \quad & \sum_{i \in \mathbb{C}} h_{i,\pi} = d_\pi \quad \forall \pi \in \mathbb{P} \quad (1) \\ (b) \quad & \sum_{\pi \in \mathbb{P}} h_{i,\pi} = K_i \quad \forall i \in \mathbb{C} \end{aligned}$$

where the constraint (a) guarantees that, for an accident happened in a given town whole the deployment plan is  $\pi$ , a EV has be allocated and the constraint (b) guarantees that each EV from a RC is allocated to an accident. This problem can be efficiently solved by a specific dual algorithm (Teghem, 2012).

#### 3.4 Model of capacity affectation under random simultaneous required EV of deployment plan

In our problem, the global number of EVs is equal to  $Q$  and the number of simultaneous required EVs for a deployment plan is not determinist. It follows a Poisson distribution (SDARC 2010, and the analysis of data issued from Toulouse SDIS). Thus, the objective is to find the best affectation of capacities which minimizes the expected value of transportation duration.

Such that after affecting the capacity and knowing simultaneous required EVs of deployment plans ( $d_\pi, \forall \pi \in \mathbb{P}$ , are crisp), the optimal transportation plan will be used.

The problem can be formulated as a nonlinear programming problem:

$$\begin{aligned} \min_{\{K_i\}_{i \in \mathbb{C}}} \quad & \mathbb{E} \left[ \min_{\{h_{i,\pi}\}_{i \in \mathbb{C}, \pi \in \mathbb{P}}} \sum_{\pi \in \mathbb{P}} \sum_{i \in \mathbb{C}} a_{i,\pi} h_{i,\pi} \right] \\ \text{s.t.} \quad & \\ (a) \quad & \sum_{i \in \mathbb{C}} h_{i,\pi} = d_\pi \quad \forall \pi \in \mathbb{P} \quad (2) \\ (b) \quad & \sum_{\pi \in \mathbb{P}} h_{i,\pi} = K_i \quad \forall i \in \mathbb{C} \\ (c) \quad & \sum_{i \in \mathbb{C}} K_i = Q \end{aligned}$$

In order to linearise the expected value, an approach based on the simulation of independent scenarios is developed.



To have a good estimation of  $\mathbb{E}[F(h_{i,\pi}, K_i)]$  (estimation with a good confidence level), a big number of scenarios ( $n$ ) is needed. To estimate  $n$ , probability theory is used (Rubinstein and Y., 1981 and Fishman, 1996).

Moreover, both decision variables  $h_{i,\pi}$  and data  $d_\pi$  depend on the scenario. Hence they become  $h_{i,\pi,s}$  and  $d_{\pi,s}$ ,  $\forall s \in \Gamma$  with  $s$  the index of the scenario and  $\Gamma$  the set of scenarios. The model (2) can be approximated by the model (3):

$$\begin{aligned} & \min_{\{K_i\}_{i \in \mathbb{C}}, \{h_{i,\pi,s}\}_{i \in \mathbb{C}, \pi \in \mathbb{P}, s \in \Gamma}} \sum_{s \in \Gamma} \sum_{\pi \in \mathbb{P}} \sum_{i \in \mathbb{C}} a_{i,\pi} h_{i,\pi,s} \\ \text{s.t.} & \\ (a) & \sum_{i \in \mathbb{C}} h_{i,\pi,s} = d_{\pi,s} \quad \forall \pi \in \mathbb{P}, \forall s \in \Gamma \\ (b) & \sum_{\pi \in \mathbb{P}} h_{i,\pi,s} = K_i \quad \forall i \in \mathbb{C}, \forall s \in \Gamma \\ (c) & \sum_{i \in \mathbb{C}} K_i = Q \end{aligned} \quad (3)$$

#### 4. RESOLUTION APPROACH

In this section, we propose an algorithm to solve the problem of affectation of capacities and to optimize an average criterion under discrete demands scenarios (see model 3). We note that in the case of the second model, scenarios of demands are generated by simulation using Poisson distribution. In fact, this problem cannot be formulated as a min-cost-flow problem, and in the literature, this problem has not still be solved. The proposed algorithm is inspired by the dual algorithm for transportation problems. To explain it, we define new variables:

- $\Delta$ : the capacity not allocated to a RC.
- $\bar{a}_{i,\pi}^s = \begin{cases} a_{i,\pi} & \text{if } h_{i,\pi,s} > 0 \\ -1 & \text{else.} \end{cases}$  the cost of using transportation arcs
- $g_{i,\pi,s}$ : the best gain to treat an accident for the deployment plan  $\pi$  by RC  $i$  in the place of another RC for the scenario  $s$ .
- $G_{i,s}$ : the best gain to increase the capacity of the RC  $i$  for the scenario  $s$ .
- $i_{i,\pi,s}^*$ : the index of the RC from which the quantity is transferred to the RC  $i$  for the scenario  $s$  and for the deployment plan  $\pi$  with the best gain
- $\pi_{i,s}^*$ : the index of the deployment plan which maximizes the gain if the transported quantity is transferred from  $i_{i,\pi,s}^*$  to  $i$
- $c^*$ : the index of the RC which the capacity have to be increased

##### 4.1 General framework

The algorithm started with the initial solution of capacity  $K_i = 0, \forall i \in \mathbb{C}$  and  $\Delta = Q$ , hence this solution does not satisfy the constraint  $\sum_{i \in \mathbb{C}} K_i = Q$ . Therefore, the initial solution which composed from transportation variables is  $h_{i,\pi,s} = 0, \forall i \in \mathbb{C}, \pi \in \mathbb{P}, s \in \Gamma$  due to the fact that the capacities of RCs are equal to zero and  $h_{e,\pi,s} = d_{\pi,s}, \forall \pi \in \mathbb{P}, s \in \Gamma$  because each accident of each deployment plan has to be satisfied (Step 0, Algorithm 1). At each iteration, capacity is transferred from  $\Delta$  to only one RC  $i, i \in \mathbb{C}$  (Step

1 to 3). The algorithm is stopped when all the capacities of  $\Delta$  are re-affected to RCs (hence, the constraint  $c$  in model 3 is satisfied, Step 4).

**Input:**  $Q$ : capacity to be affected,  $a_{i,\pi}$ : lost time to satisfy an accident for the deployment plan  $\pi$  by a EV from the RC  $i$ ,  $d_{\pi,s}$ : number of accidents of the deployment plan  $\pi$  of the scenario  $s$ .

**Output:** An optimal capacity allocation  $K_i$  and a transportation solution  $h_{i,\pi,s}$  for each scenario  $s$ .

**Step 0:**  $K_i := 0, \forall i \in \mathbb{C}$ ,  $\Delta := Q$ ,  $h_{i,\pi,s} = 0, \forall i \in \mathbb{C}, \forall \pi \in \mathbb{P}, \forall s \in \Gamma$ ,  $h_{e,\pi,s} = d_{\pi,s}, \forall \pi \in \mathbb{P}, \forall s \in \Gamma$ .

**Step 1:** Find the best RC to increase its capacity ( $c^*$ ).

**Step 2:** Compute the maximal value of capacity to transfer from  $\Delta$  to  $c^*$ .

**Step 3:** Update the solution ( $K_i$  and  $h_{i,\pi,s}$ ) and  $\bar{a}_{i,\pi}^s$ .

**Step 4:** If  $\Delta > 0$  then go to Step 1 else STOP.

Algorithm 1. Optimal affectation of capacity

Therefore we are forced, at each iterations, to answer two questions: to which RC the capacity have to be transferred (Step 1)? and how many capacities (Step 2)?

##### 4.2 Research of the best RC: $c^*$

To answer to the first question, we have to evaluate the gain when the capacity of RCs is increased. Noted that, increasing capacity of a given RC means that this center has EVs which can be assigned to a deployment plan. So it is more interesting to treat accidents happened in towns, which have the same deployment plan, by this RC and not by the previous one. More formally, the equation (4) is used to compute the gain of each deployment plan (for a given RC  $i$  and a given scenario  $s$ ) and to choose the best one (equation 5, the symbol  $\boxtimes$  means that they are not a more interesting center to treat accidents).

$$g_{i,\pi,s} = \max_{j \in \mathbb{C}_i^+} (0, \bar{a}_{j,\pi}^s - a_{i,\pi} : \bar{a}_{j,\pi}^s \neq -1)$$

with

$$\mathbb{C}_i^+ = \mathbb{C} \cup \{e\} \setminus i \quad (4)$$

$$i_{i,\pi,s}^* = \begin{cases} \arg \max_{j \in \mathbb{C}_i^+} (0, \bar{a}_{h,\pi}^s - a_{i,\pi} : \bar{a}_{h,\pi}^s \neq -1) & \text{if } g_{i,\pi,s} > 0 \\ \boxtimes & \text{else.} \end{cases} \quad (5)$$

The gain obtained by increasing the capacity of a RC  $i$  for a scenario  $s \in \Gamma$  is the best transfer whatever the deployment plan  $\pi \in \mathbb{P}$ , so it can be computed using equation (6). Moreover, we have to keep the index of the optimal transfer to update the transportation variables. The optimal transfer is the one which has the better gain so the one which maximizes the difference between transport quantity from ( $i_{i,\pi,s}^*$  to  $\pi_{i,s}^*$ ) and transport quantity from  $i$  to  $\pi_{i,s}^*$ , with  $\pi_{i,s}^*$  defined in equation 7 (the symbol  $\boxtimes$  means that they are not a more interesting transfer).

$$G_{i,s} = \max_{\pi \in \mathbb{P}} (g_{i,\pi,s}) \quad (6)$$

$$\pi_{i,s}^* = \begin{cases} \arg \max_{\pi \in \mathbb{P}} (g_{i,\pi,s}) & \text{if } G_{i,s} > 0 \\ \boxtimes & \text{else.} \end{cases} \quad (7)$$

The gain  $G_{i,s}$  means that if the capacity of the RC  $i$  is increased by a value  $y$ , the cost function of previous solution decreases by  $y \times \sum_{s \in \Gamma} G_{i,s}$ . The best RC  $c^*$  which increases the capacity is the one which maximizes the gain (equation 8 and 9)

$$GT = \max_{i \in \mathbb{C}} \left( \sum_{s \in \Gamma} G_{i,s} \right) \quad (8)$$

$$c^* = \begin{cases} \arg \max_{i \in \mathbb{C}} \left( \sum_{s \in \Gamma} G_{i,s} \right) & \text{if } GT > 0 \\ \boxtimes & \text{else.} \end{cases} \quad (9)$$

$c^* = \boxtimes$  arrives when  $\forall s \in \Gamma$  and  $G_{i,s} = 0$ . So,  $c^* = \boxtimes$  means that the function can be minimized by adding capacity. In other words, we have too much capacity. The excess of capacity is transferred by default to first one RC,  $i = 1$  (equation 10 of section 4.3).

#### 4.3 Evaluation of maximal quantity to transfer

We have answered to the first question by choosing the RC  $c^*$ . Now, the maximal capacity than can be transfer from  $\Delta$  to  $c^*$  with the same gain has to be find. The gain depends on the accident transferred from the RC  $i_{c^*,\pi,s}^*$  to the RC  $c^*$ . Then, if this transfer is made for all scenarios, the gain is constant. Hence the maximal transfer is computed using equation (10):

$$T_{c^*} = \min_{s \in \Gamma} (\Delta, h_{i_{c^*,\pi_{c^*,s}^*}^*, \pi_{c^*,s}^*, s}^* : i_{c^*,\pi_{c^*,s}^*}^*, \pi_{c^*,s}^*, s \neq \boxtimes) \quad (10)$$

Noted that,  $c^* = \boxtimes$  arrives when  $G_{i,s} = 0, \forall s \in \Gamma$ . So, all  $\pi_{i,s}^* = \boxtimes$  (equation 7) and all  $i_{i,\pi,s}^* = \boxtimes$  (equation (5)). Then  $\nexists h_{i_{c^*,\pi_{c^*,s}^*}^*, \pi_{c^*,s}^*, s}^*$  such that  $i_{c^*,\pi_{c^*,s}^*}^*, \pi_{c^*,s}^*, s \neq \boxtimes$ .

Hence  $T_{\boxtimes} = \Delta$ .

Finally, we actualize the current solution:

- $K_{c^*} := K_{c^*} + T_{c^*}$
- $\Delta := \Delta - T_{c^*}$
- $h_{i_{c^*,\pi_{c^*,s}^*}^*, \pi_{c^*,s}^*, s}^* := h_{i_{c^*,\pi_{c^*,s}^*}^*, \pi_{c^*,s}^*, s}^* - T_{c^*}, \forall s \in \Gamma$
- $h_{c^*,\pi_{c^*,s}^*}^* := h_{c^*,\pi_{c^*,s}^*}^* + T_{c^*}, \forall s \in \Gamma$

**Remark:** the solutions are integers:  $K_i, h_{i,\pi,s} \in \mathbb{N}, \forall i \in \mathbb{C}, \pi \in \mathbb{P}, s \in \Gamma$  if and only if  $d_{\pi,s} \in \mathbb{N}, \forall \pi \in \mathbb{P}, s \in \Gamma$  and  $Q \in \mathbb{N}$ .

### 5. ILLUSTRATION OF ALGORITHM

In this section we illustrate the algorithm, to facilitate understanding of the algorithm we represent the data in table 1 where a first part of the table represent the penalty coefficient (the column are the deployment plan and the line the RC), the last column is affectation of capacity ( $K_i, \Delta$ ) and the last line of the table are the scenarios of demand.

**Remark:** When simultaneous requirements follow a Poisson distribution the scenarios of demand are simulated using this distribution.

	$\pi = 1$	$\pi = \dots$	$\pi = P$	$K_i$
$i = 1$	$a_{1,1}$	$a_{1,\dots}$	$a_{1,P}$	$\dots$
$i = \dots$	$a_{\dots,1}$	$a_{\dots,\dots}$	$a_{\dots,P}$	$\dots$
$i = C$	$a_{C,1}$	$a_{C,\dots}$	$a_{C,P}$	$\dots$
$e$	$a_{e,1}$	$a_{e,\dots}$	$a_{e,P}$	$\Delta = \dots$
$d_{s=1}$	$d_{1,1}$	$d_{\dots,1}$	$d_{P,1}$	
$d_{s=\dots}$	$d_{1,\dots}$	$d_{\dots,\dots}$	$d_{P,\dots}$	
$d_{s=S}$	$d_{1,S}$	$d_{\dots,S}$	$d_{P,S}$	

Table 1. general table

For simplicity of illustration, we apply the algorithm to a example with 2 scenario of demand, 3 deployment plans, 3 RCs and  $Q = 20$ . The data are given in the table (2). Let  $M$  a big number. Moreover, the initial solution of transportation quantity from RC which is different than zero, is presented in the table (2).  $M_{(4)}$  means that  $h_{e,1,1} = 10$  and  $h_{e,1,2} = 4$ .

	$\pi = 1$	$\pi = 2$	$\pi = 3$	$K_i$
$i = 1$	0	10	2	0
$i = 2$	4	0	3	0
$i = 3$	15	1	0	0
$e$	$(10)M_{(4)}$	$(2)M_{(8)}$	$(14)M_{(6)}$	$\Delta = 20$
$d_{s=1}$	10	2	14	
$d_{s=2}$	4	8	6	

Table 2. data and initial solution

First we compute the gain to increase the capacity for each deployment plan  $\pi$  of each RC  $i$ , since at the beginning only the RC  $e$  satisfies the demand ( $h_{e,\pi,s} = d_{\pi,s}$ ). The gains for  $i = 1$  and  $s = 1$  are  $g_{1,1,1} = (M - 0)$  (for  $\pi = 1$ ),  $g_{1,2,1} = (M - 10)$  and  $g_{1,3,1} = (M - 2)$ . So  $i_{1,\pi,1}^* = e, \forall \pi \in \mathbb{P}$  and  $G_{i,s} = M$  with  $\pi_{i,s}^* = 1$ . We repeat this for all scenarios and for others RCs. Then we obtain:  $\sum_{s \in \Gamma} G_{i,s} = 2M, \forall i \in \mathbb{C}$ . Hence, we choose arbitrary the RC  $c^* = 1$ . The maximal capacity which can be transferred with this gain is the minimal quantity between  $h_{e,1,1} = 10, h_{e,1,2} = 4$  and  $\Delta = 20$  hence  $T_1 = 4$ . To finish this iteration, we update the initial solution (see Table 3).

	$\pi = 1$	$\pi = 2$	$\pi = 3$	$K_i$
$i = 1$	$(4)0_{(4)}$	10	2	4
$i = 2$	4	0	3	0
$i = 3$	15	1	0	0
$e$	$(6)M_{(0)}$	$(2)M_{(8)}$	$(14)M_{(6)}$	$\Delta = 16$
$d_{s=1}$	10	2	14	
$d_{s=2}$	4	8	6	

Table 3. data

The gain of the first RC becomes  $\sum_{s \in \Gamma} G_{1,s} = (M - 0) + (M - 2)$  since  $h_{e,1,2} = 0$  and  $h_{e,2,2}$  and  $h_{e,3,2} > 0$ . But others RCs still have a gain of  $2M$ . So we increase the capacity of RC 2 and then 3, with  $T_2 = 2$ , and  $T_3 = 6$ . The result of these two iterations is given in the Table (4).

	$\pi = 1$	$\pi = 2$	$\pi = 3$	$K_i$
$i = 1$	$(4)0_{(4)}$	10	2	4
$i = 2$	4	$(2)0_{(2)}$	3	2
$i = 3$	15	1	$(6)0_{(6)}$	6
$e$	$(6)M_{(0)}$	$(0)M_{(6)}$	$(8)M_{(0)}$	$\Delta = 8$
$d_{s=1}$	10	2	14	
$d_{s=2}$	4	8	6	

Table 4. Results iteration 1

First we compute the gain to increase the capacity for each deployment plan  $\pi$  of each RC  $i$ . The best gains are:  $g_{1,1,1} = (M - 0)$  for RC 1 and scenario 1 with  $\pi_{1,1}^* = 1$ ,  $g_{1,2,2} = (M - 10)$  for RC 1 and scenario 2 with  $\pi_{1,2}^* = 2$ ,

$g_{2,3,1} = (M - 3)$ ,  $g_{2,2,2} = (M - 0)$ ,  $g_{3,3,1} = (M - 0)$  and  $g_{3,2,2} = (M - 1)$ . So we have  $\sum_{s \in \Gamma} G_{1,s} = 2M - 10$ ,  $\sum_{s \in \Gamma} G_{2,s} = 2M - 3$  and  $\sum_{s \in \Gamma} G_{3,s} = 2M - 1$ . The RC to be increased is  $c^* = 3$ . The maximal capacity which can be transferred with this gain is the minimal quantity between  $h_{e,3,1} = 8$ ,  $h_{e,2,2} = 6$  and  $\Delta = 8$  hence  $T_3 = 6$ . To finish this iteration, we update the initial solution (see Table 5).

	$\pi = 1$	$\pi = 2$	$\pi = 3$	$K_i$
$i = 1$	$(4)0_{(4)}$	10	2	4
$i = 2$	4	$(2)0_{(2)}$	3	2
$i = 3$	15	$(0)1_{(6)}$	$(12)0_{(6)}$	12
$e$	$(6)M_{(0)}$	$(0)M_{(0)}$	$(2)M_{(0)}$	$\Delta = 2$
$d_{s=1}$	10	2	14	
$d_{s=2}$	4	8	6	

Table 5. Results iteration 4

We repeat the same thing than the previous iteration. The temporally results are:

- The best gains are:  $g_{1,1,1} = M$ ,  $g_{1,\emptyset,2} = 0$ ,  $g_{2,3,1} = M - 3$ ,  $g_{2,2,2} = 1$ ,  $g_{3,3,1} = M$  and  $g_{3,\emptyset,2} = 0$
- So we have  $\sum_{s \in \Gamma} G_{1,s} = M$ ,  $\sum_{s \in \Gamma} G_{2,s} = M - 2$  and  $\sum_{s \in \Gamma} G_{3,s} = M$   $T_1 = \min(h_{e,1,1} = 6, \Delta = 2) = 2$

Since  $\Delta = 0$  we update the solution (see Table 6) and the algorithm stops.

	$\pi = 1$	$\pi = 2$	$\pi = 3$	$K_i$
$i = 1$	$(6)0_{(4)}$	10	2	6
$i = 2$	4	$(2)0_{(2)}$	3	2
$i = 3$	15	$(0)1_{(6)}$	$(12)0_{(6)}$	12
$e$	$(4)M_{(0)}$	$(0)M_{(0)}$	$(2)M_{(0)}$	$\Delta = 0$
$d_{s=1}$	10	2	14	
$d_{s=2}$	4	8	6	

Table 6. Final results

## 6. CONCLUSION

This paper deals with the affectation of EVs to RCs under random demand. The aim is to determinate optimal capacity of each RC to minimize the expected value of transportation duration. A tactical aggregated model and an algorithm are proposed. The deployment plans are supposed fixed. Both the total number of EVs and the set of RCs are known. An example is given to illustrate the resolution method.

In the real world an initial solution of capacity affectation is done. Indeed it is not truly realistic to determine these capacities from scratch. A high modification of this affectation impacts fire fighting units (professionals as well as volunteers) and it is expensive to relocated EVs. Thus, our future work will focus on the modification of the current affectation of capacities under random demand and relocation costs. Costs which take into account the mobility of Fireman and the transfer of EVs from RC  $i$  to  $j$  will be integrated. The main objective will be the use of these techniques to help the SDIS to optimize both deployment plans and RCs capacities.

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